Robust adaptive finite element schemes for nonlinear viscoelastic solid deformation: an investigative study

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1 Phase II research

This report outlines the seed project work during December 1997 to March 1998. During this period work at BICOM provided adaptive error estimates in weak norms for Volterra problems which apply to creep and relaxation. This and the work carried out during the first phase can be found in [2], [3] and [1], and has now been submitted for publication in research journals. Another paper, [4], is expected to be submitted before the summer.

Prof. Whiteman and Dr. Shaw spent much of this second phase examining the connection between the **hereditary integral** and **internal variable** models of viscoelasticity. As a result it now seems feasible to establish an *a posteriori* bound on the difference between the numerical solutions that result from each of these algorithms. This is potentially important since (using the triangle inequality) this will mean that the space-time *a posteriori* error estimates in [2], [3] and [4] can be used to generate internal variable algorithms, for two- or three-dimensional (quasistatic linear) viscoelasticity, that are adaptive in both space and time.

1.1 Adaptivity

In February Dr A R Johnson transmitted to BICOM an ARL code which performs finite element analyses of viscoelastic beam deformation, including dynamic effects. This code employs Dr Johnson's internal variable formulations to simulate the dynamic response of a cantilevered beam. It was compiled and ran immediately at BICOM.

When Dr Johnson arrived at BICOM, for his 1-8 March 1998 visit, it was agreed that we should jointly build on BICOM's capability in adaptive finite element methods to extend the ARL code and apply it to creep problems. If successful this might lead to the use of the adaptive methods in the context of dynamic viscoelastic deformation. The installation of the adaptivity was successfully undertaken for a problem involving the quasistatic creep of a viscoelastic cantilevered beam under the action of a piecewise linear vertical force applied to the free end.

The original effort to include the adaptivity for the dynamic response of the beam was abandoned. Initial trials in this direction by Drs. Johnson and Shaw indicate that the adaptive numerical simulation of dynamic effects will require more research. This is consistent with the view taken in the seed project proposal where work on dynamic problems was postponed to subsequent years of the research.

1.2 Internal Variable Formulations for Nonlinear Viscoelasticity

For linear viscoelastic problems the history integral formulation used by BICOM and the internal variable formulations used by ARL, assuming the use of the same Prony-series approximations for the viscoelastic relaxation functions, lead to identical analytical solutions. This fact is the motivation behind the aim of using, in the nonlinear viscoelastic context, the existing internal variable formulations to provide a general theory for devising reduced time functions which are required for nonlinear viscoelastic constitutive relations. It should be remembered (see the original proposal) that BICOM proposed a nonlinear history integral which successfully modelled the behaviour of Nylon 6,6 in creep. However, the reduced time depends on some function of stress or strain or both, and the work so far has provided only a function of stress which works for a particular Nylon context. We are, therefore, considering the alternative approach in which the strain energy density is split into two parts; one arising from nonlinear elasticity, and another arising from the viscous effects.

This approach to treating the viscous effects results in a system of nonlinear ordinary differential equations for determining the viscous stresses. In view of the complimentary expertise in ARL and BICOM it is proposed that the application of the splitting approach with internal variables to the Nylon 6,6 problem previously treated by BICOM be investigated during the remainder of the first year of the project.

1.3 Other details

- After discussion with Dr. Johnson during his visit to BICOM it was agreed that large scale computing projects, such as implementing the results in [2], [3] and [4], should be given a lower priority than continued theoretical work on viscoelastic modelling. This view is to some extent justified by the success of implementing the adaptivity into the beam code. Once the physical models are understood computation is mostly a matter only of time and effort—what is important is that this time and effort be directed toward the correct goals.
- Prof. Whiteman and Dr. Shaw are planning to visit Dr. Johnson at NASA Langley in late summer 1998 (precise dates to be confirmed). This will give them the opportunity of beginning the computational work on the associated problem of non-Fickian diffusion in polymers (which is separately funded), and it is hoped that many software components arising from this work will feed directly into the ARL project.

2 Year two of the Project

It remains the hope at BICOM that a second year of this project will be funded. This will enable the above nonlinear work to be applied in the viscodynamic context as set out in Section 4.3 of the original proposal.

3 Administrative actions and other details

- During the second phase Dr. Shaw has been supported in part from this contract, and will continue to be supported in part for the duration.

- Professor Whiteman was an invited speaker at the SACAM (South African Conference on Applied Mechanics) in Cape Town in January 1998, where he lectured on the research financed in part by this project.
- Prof. Whiteman and Dr. Shaw will each give a twenty-minute presentation of work related to this seed project during the **Solid Mechanics and Computational Solid Mechanics** mini-symposium, at the British Applied Mathematics Colloquium to be held at Brunel University during 6–9 April 1998.
- Dr. Shaw has been invited to give research seminars at Brunel on 13 March, and at Leicester University on 14 May. The subject of these talks will be the work completed during the first phase of this seed project.
- The second receipt of funds from this contract will result from the submission of this second interim report.

References

[1] Simon Shaw and J. R. Whiteman. Negative norm error control for second-kind convolution Volterra equations. BICOM Technical Report 98/6, see `http://www.brunel.ac.uk/~icsrbicm`, 1998.

[2] Simon Shaw and J. R. Whiteman. Numerical solution of linear quasistatic hereditary viscoelasticity problems I: *a priori* estimates. BICOM Technical Report 98/2, see `http://www.brunel.ac.uk/~icsrbicm`, 1998.

[3] Simon Shaw and J. R. Whiteman. Numerical solution of linear quasistatic hereditary viscoelasticity problems II: *a posteriori* estimates. BICOM Technical Report 98/3, see `http://www.brunel.ac.uk/~icsrbicm`, 1998.

[4] Simon Shaw and J. R. Whiteman. Numerical solution of linear quasistatic hereditary viscoelasticity problems III: *a posteriori* estimates in a weak norm. BICOM Technical Report 98/4, see `http://www.brunel.ac.uk/~icsrbicm`, 1998.